Sirindhorn International Institute of Technology
Thammasat University at Rangsit
School of Information, Computer and Communication Technology

TCS 455: Problem Set 6

Semester/Year: **2/2009**
Course Title: Mobile Communications
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Due date: 12PM, Feb 8, 2010 (not due for those in TCS455)

1. Recall that the baseband OFDM modulated signal can be expressed as

$$s(t) = \sum_{k=0}^{N-1} S_k \frac{1}{\sqrt{N}} 1_{[0,T_s]}(t) \exp\left(j \frac{2\pi kt}{T_s} \right)$$

where $S_0, S_1, \ldots, S_{N-1}$ are the (potentially complex-valued) messages.

Let $T_s = 1$ [ms], *N* = 8, and

$$\left(S_0, S_1, \ldots, S_{N-1} \right) = \left(1-j, 1+j, 1, 1-j, -1-j, 1, 1-j, -1+j \right)$$

a. Use MATLAB to plot the following waveforms as accurately as you can.

i. $a(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \operatorname{Re}\{S_k\} \cos\left(\frac{2\pi kt}{T_s} \right)$

ii. $b(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \operatorname{Im}\{S_k\} \sin\left(\frac{2\pi kt}{T_s} \right)$

iii. $\operatorname{Re}\{s(t)\}$

b. What is the relationship between $a(t)$, $b(t)$, and $\operatorname{Re}\{s(t)\}$?

2. Consider the discrete-time complex FIR channel model

$$y[n] = \{h * x\}[n] + w[n] = \sum_{m=0}^{2} h[m]x[n-m] + w[n]$$

where $w[n]$ is zero-mean additive Gaussian noise.

In this question, assume that *h*[*n*] has unit energy and that $H(z)$ has two zeros at $z_1 = \rho e^{j\frac{2\pi}{3}}$ and $z_2 = \frac{1}{\rho}$ where $\rho < 1$.

a. Plot $\left|H\left(e^{j\omega}\right)\right| = \left|H(z)\big|_{z=e^{j\omega}}\right|$ in the range $\omega = 0:\frac{2\pi}{80}:2\pi$ for ρ = 0.5 and 0.99

b. For OFDM system with block size *N* = 8, find the corresponding channel gains $H_k = H(z)\big|_{z=e^{j\frac{2\pi}{N}k}}$, *k* = 0, 1, 2, …, *N*-1 for ρ = 0.5 and 0.99. In particular, complete the following table.

Ch # k	$\rho = 0.5$		$\rho = 0.99$	
	$H\left(e^{j\frac{2\pi}{N}k}\right)$	$\left\|H\left(e^{j\frac{2\pi}{N}k}\right)\right\|$	$H\left(e^{j\frac{2\pi}{N}k}\right)$	$\left\|H\left(e^{j\frac{2\pi}{N}k}\right)\right\|$
0				
1				
2				
3				
4				
5				
6				
7				

3. OFDM simulation: Write a MATLAB code to perform the following operations
   a. Generate 10,000 OFDM blocks, each is an 8 dimensional QPSK vector. Each element of the vector is independently and randomly chosen from the constellation set $M = \{1+j, 1-j, -1+j, -1-j\}$.
   b. Perform the IFFT to each vector.
   c. Add the cyclic prefix to each block and transmit over the FIR channel defined in the previous question. Assume $w[n] \equiv 0$. Consider two cases: ρ = 0.5 and 0.99.
   d. At the receiver, remove the cyclic prefix and perform the FFT to get R_k.
   e. <u>**Detect**</u> the transmitted symbols at each channel. Use the ML (maximum likelihood) detector. <u>**Record**</u> the symbol error rates (SER) for each channel. (All of them should be 0 in this question.)

      Hint: When there is no noise, you have $R_k = H_k S_k$. Therefore, $S_k = \frac{R_k}{H_k}$.

When there is some zero-mean Gaussian noise, $R_k = H_k S_k + W_k$ where W_k is the noise (in the frequency domain). Therefore,

$$\frac{R_k}{H_k} = S_k + \underbrace{\frac{W_k}{H_k}}_{\text{New Noise}} .$$

Because the noise is Gaussian and zero-mean, the noise will most likely not take $\frac{R_k}{H_k}$ too far from S_k. Therefore, the ML detector gives

$$\hat{S}_k = \arg\min_{s \in M} \left\{ \left\| R_k - sH_k \right\| \right\} = \arg\min_{s \in M} \left\{ \left\| \frac{R_k}{H_k} - s \right\| \right\},$$

i.e. it detects S_k as the *__closest__* message *s* in the constellation to $\frac{R_k}{H_k}$. Of course, the noise can be large and shift $\frac{R_k}{H_k}$ too far from the original S_k. Therefore, $\hat{S}_k$ may be different from S_k. This is when symbol error occurs.

4. Repeat Question 3. However, in this question, the channel noise is non-zero. $w[n]$ is now i.i.d. complex-valued Gaussian noise. Its real and imaginary parts are i.i.d. zero-mean Gaussian with variance $N_0/2$ where

$$N_0 = \frac{2}{3\text{SNR}_{\text{norm}}}.$$

Assume SNR_{norm} is 2 dB.

   a. Complete the following table.

Ch # k	$\rho = 0.5$	$\rho = 0.99$
0		
1		
2		
3		
4		
5		
6		
7		

   b. Explain your SER results using the table in Question 2.